**Al-Farabi KAZAKH NATIONAL UNIVERSITY**

**Faculty of mechanics and mathematics**

**Educational program for the specialty «5В060100-Mathematics»**

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|  | Approved by the Faculty Scientific Council meetingProtocol №\_\_\_ from \_\_\_\_\_\_\_\_\_\_\_\_ 2012 **Dean of the faculty**  **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_D.Zh. Akhmed-Zaki** |

**SULLABUS**

**по основному обязательному / основному элективному / профессиональному элективному**

**module \_\_\_ «\_\_\_\_\_\_\_\_\_\_\_\_\_»** \_\_\_ credits

**includes courses**

**«\_\_\_\_\_\_\_\_\_» - «Differentiation and optimization»** (2 credits)

1 course, k/s (r/s), autumn semester

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**ПАСПОРТ модуля:**

▪ **Цель** (единая для модуля, формулируется в соответствии с названием модуля и с представленными синтезированными целями дисциплин, входящих в модуль).

▪ **Задачи**: (единым списком, объединяющим задачи по дисциплинам, входящим в модуль, задачи должны обязательно быть направлены на формирование компетенций, сформулированных в Спецификации).

▪ **Результаты обучения** по модулю (объединенные результаты дисциплин в системе компетенций, см. Спецификацию).

- Общие компетенции:

инструментальные: … *(кратко в виде аннотации, 2-3 предложения)*;

межличностные: … *(кратко в виде аннотации, 2-3 предложения)*;

системные: … *(кратко в виде аннотации, 2-3 предложения)*;

- Предметные компетенции: … *(кратко в виде аннотации, 2-3 предложения)*.

▪ **Пререквизиты, постреквизиты**.

**I course «\_\_\_\_\_\_\_\_» - «Differentiation and optimization»** (2 credits)

**PASSPORT of the course:**

**Aim** (in concordance with the aim of the module).

Using of the differentiation theory for solving different optimization control problems.

**Problems:**

Determination of different forms of derivatives for functions, functionals, operators. Analysis relations between optimization problems and necessary conditions of optimality. Proof of different forms of necessary conditions of optimality.

**Results:**

General optimization control methods for different problems.

**Competences**:

Minimization of functions and functionals. Variations calculus. Optimization control problems for the systems described by ordinary differential equations. Optimization control problems for distributed systems.

**STRUCTURE, VOLUME AND CONTENT OF THE COURSE**

|  |  |  |  |
| --- | --- | --- | --- |
| **week** | **Course «\_\_» - «Differentiation and optimization»,** 3 credits | | |
| **subject** | **hours** | **tasks** |
| **Part I. Minimization of functionals** | | | |
| **1** | **Lecture 1** «Introduction» | **2** | Give example of functions with Fermat condition that has one, two and three solution. |
| **2** | **Lecture 2 «**Fermat condition. Euler equation. Dirichlet integral**»**  **Practical work 2 «**ExampleofFermat condition and Euler equation**»** | **1**  **1** |
| **3** | **Lecture 3 «**Gateaux derivative. Definition. Examples**»**  **Practical work 3 «**Finding of Gateaux derivatives**»** | **1**  **1** |  |
| **4** | **Lecture 4 «**Gateaux derivative. Definition. Examples**»**  **Practical work 4 «**Finding of Gateaux derivatives**»** | **1**  **1** | Find Gateaux derivatives of the functional *I* in the space *V*: |
| **5** | **Lecture 5 «**Obtaining of stationary condition**»**.  **Practical work 5 «**Obtaining of stationary condition. Examples**»** | **1**  **1** | Determine stationary conditions for the functionals from the previous task. |
| **6** | **Lecture 6 «**Problems with convex constraints. Necessary conditions of optimality.Variational inequality**»**  **Practical work 6 «**Examples of variational inequalities**»** | **1**  **1** | Determine variational inequalities for the functionals from the previous tasks with control determined on the unit interval. |
| **7** | **Lecture 7 «**Minimization of functionals on subspace**»**  **Practical work 7 «**Examples of minimization of functionals on subspaces**»** | **1**  **1** | Consider a problem of the minimization of the function    On the subspace    where *a*, *b*, *c* are parameters. Task:   1. Find Gateaux derivative. 2. Find stationary condition. 3. Find annulator of the set *U*. 4. Find the solution with using of annulator. 5. Find the derivative with respect to the subspace. 6. Find the solution with using of the derivative with respect to the subspace.   Variants: |
| **7** | **Control 1** |  |  |
| **Part II. Optimization control problems** | | | |
| **8** | **Lecture 8 «**Necessary results of functional analysis**»**  **Practical work 8 «**Finding of Gateaux derivatives for operators**»** | **1**  **1** | Find Gateaux derivatives for operators:   * Family of functions (Jacobian). * Degree operator in the space of integrable functions. |
| **9** | **Lecture 9 «**Abstract linear systems. Necessary conditions of optimality**»**  **Practical work 9 «**Abstract linear systems. Necessary conditions of optimality**»** | **1**  **1** |  |
| **10** | **Lecture 10 «**Linear elliptic equations. Necessary conditions of optimality**»**  **Practical work 10 «**Abstract linear elliptic equations. Necessary conditions of optimality**»** | **1**  **1** | Find the solution of the minimization problem for the functional    on the set *U*. Variants:   1. , 2. , 3. , |
| **11** | **Lecture 11 «**Inverse function theorem**»**  **Practical work 11 «**Applications of the Inverse function theorem**»** | **1**  **1** | Find the derivative in the zero of the inverse function *f* -1, where the function *f* is known:  1.  2.  3. |
| **12** | **Lecture 12** «Abstract nonlinear systems. Necessary conditions of optimality»  **Practical work 12** «Abstract nonlinear systems. Necessary conditions of optimality» | **1**  **1** |  |
| **13** | **Lecture 13 «**Nonlinear elliptic equations**»**  **Practical work 13 «**Example of nonlinear elliptic equations» | **1**  **1** | Determine functional spaces for boundary problems: |
| **14** | **Lecture 14 «**Sobolev embedding theorem**»**  **Practical work 13 «**Application of Sobolev embedding theorem**»** | **1**  **1** | By Sobolev theorem the space  is the subspace of , where *q* satisfies the equality  if  and arbitrary if . If , then each function from  is continuous. Questions:   1. What is an order of integrability *р* for continuity of functions from  for different value *n* of dimension of the set Ω? 2. What is the value *m* for continuity of functions from  for different value *n* of dimension of the set Ω? 3. What is the value of dimension, for enclosure the space  to the space  for different values of *р*? |
| **15** | **Lecture 15 «**Necessary conditions of optimality for nonlinear elliptic equations**»**  **Practical work 15 «**Example of necessary conditions of optimality for nonlinear elliptic equations**»** | **1**  **1** | Analyze the following boundary problems: |
| **15** | **Control 2** |  |  |

**Key words**

Optimization control problem. Necessary condition of optimality. Gateaux derivative. Elliptic equations. Sobolev space. Sobolev embedding theorem.

**Literature**

**General**

1. Серовайский С.Я. Оптимизация и дифференцирование. – Алматы, Print-S, 2006.
2. Serovajsky S. Practical Course of the Optimal Control Theory with Examples. Алматы, Қазақ университеті, 2011.

**Additional**

1. Алексеев В. М., Тихомиров В. М., Фомин С. В. Оптимальное управление. – М., Наука, 1979. – 432 с.
2. Васильев Ф. П. Методы решения экстремальных задач. – М.: Наука, 1981. – 400 с.
3. Габасов Р., Кириллова Ф. Качественная теория оптимальных процессов. – М., Наука, 1907. – 507 с.
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5. Канторович Л. В., Акилов Г. П. Функциональный анализ. – М., Наука, 1977. – 744 с.
6. Лионс Ж.-Л. Оптимальное управление системами, описываемыми уравнениями с частными производными. – М., Мир, 1972. – 492 с.
7. Лионс Ж.-Л. Некоторые методы решения нелинейных краевых задач. – М., Мир, 1972. – 576 с.
8. Обен Ж. П., Экланд И. Прикладной нелинейный анализ. – М., Мир, 1988. – 510 с.

**Tasks and methodical guidelines for individual student work**

1) Find Gateaux derivatives of the functional *I* in the space *V*.

1. 
2. 
3. 

2) Consider a problem of the minimization of the function



On the subspace



where *a*, *b*, *c* are parameters. Task:

1. Find Gateaux derivative.
2. Find stationary condition.
3. Find annulator of the set *U*.
4. Find the solution with using of annulator.
5. Find the derivative with respect to the subspace.
6. Find the solution with using of the derivative with respect to the subspace.

Variants:

1. 
2. 
3. 

3) Find the solution of the minimization problem for the functional



on the set *U*. Variants:

1. , 
2. , 
3. , 

4) Find Gateaux derivatives for operators:

* Family of functions (Jacobian).
* Degree operator in the space of integrable functions.

5) Find the derivative in the zero of the inverse function *f* -1, where the function *f* is known:

1. 

2. 

3. 

Determine functional spaces for boundary problems:

1. 
2. 
3. 

6) By Sobolev theorem the space  is the subspace of , where *q* satisfies the equality  if  and arbitrary if . If , then each function from  is continuous. Questions:

1. What is an order of integrability *р* for continuity of functions from  for different value *n* of dimension of the set Ω?
2. What is the value *m* for continuity of functions from  for different value *n* of dimension of the set Ω?
3. What is the value of dimension, for enclosure the space  to the space  for different values of *р*?

7) Analyze the following boundary problems:

1.  
2.  
3.  

8) Determine necessary conditions of optimality and fine the control:

1.  





1.  





1.  





**Form of control and competence**

Tasks for individual student works.

Control – 2 times.

Exam during the session.

**Criterion for the estimate of the knowledge**

Individual student works – 60 %

Examination – 40 %

**Consultations are taken during the office-hours of the lecturer and by mail.**

**Шкала оценки знаний:**

|  |  |  |  |
| --- | --- | --- | --- |
| Оценка по буквенной системе | Цифровой эквивалент баллов | %-ное содержание | Оценка по традиционной системе |
| А | 4,0 | 95-100 | Отлично |
| А- | 3,67 | 90-94 |
| В+ | 3,33 | 85-89 | Хорошо |
| В | 3,0 | 80-84 |
| В- | 2,67 | 75-79 |
| С+ | 2,33 | 70-74 | Удовлетворительно |
| С | 2,0 | 65-69 |
| С- | 1,67 | 60-64 |
| D+ | 1,33 | 55-59 |
| D- | 1,0 | 50-54 |
| F | 0 | 0-49 | Неудовлетворительно |
| I  (Incomplete) | - | - | «Дисциплина не завершена»  (*не учитывается при вычислении GPA)* |
| P  (Pass) | **-** | **-** | «Зачтено»  (*не учитывается при вычислении GPA)* |
| NP  (No Рass) | **-** | **-** | «Не зачтено»  (*не учитывается при вычислении GPA)* |
| W  (Withdrawal) | - | - | «Отказ от дисциплины»  (*не учитывается при вычислении GPA)* |
| AW  (Academic Withdrawal) |  |  | Снятие с дисциплины по академическим причинам  (*не учитывается при вычислении GPA)* |
| AU  (Audit) | - | - | «Дисциплина прослушана»  (*не учитывается при вычислении GPA)* |
| Атт. |  | 30-60  50-100 | Аттестован |
| Не атт. |  | 0-29  0-49 | Не аттестован |
| R (Retake) | - | - | Повторное изучение дисциплины |

**Политика академического поведения и этики**

Будьте толерантны, уважайте чужое мнение. Возражения формулируйте в корректной форме. Плагиат и другие формы нечестной работы недопустимы. Недопустимы подсказывание и списывание во время сдачи СРС, промежуточного контроля и экзамена, копирование решенных задач другими лицами, сдача экзамена за другого студента. Студент, уличенный в фальсификации любой информации курса, получит итоговую оценку «F».

*Рассмотрено на заседании кафедры*

*протокол № \_\_ от « \_\_ » \_\_\_\_\_\_\_\_\_\_\_ 2012 г.*

**Зав.кафедрой**

**Лектор**